2009 MathPath Courses and Descriptions

Descriptions are given by week, first breakouts and then plenaries.

Concrete Groups. Introduction to group theory through symmetry groups of physical objects.

MathCounts. Alan Lippert (faculty member and MathPath Treasurer) was earlier the coach of the Washington State MathCounts team and coach of a national MathCounts winner. He has thousands of collected problems of varying difficulty and proven coaching techniques. He gives courses both at the Regional Level, for those trying to make a state team, and National Level, for those who have made the state team and what to excel at nationals.

Induction. The standard mathematical technique, but the problems are chosen to be accessible to these students, say by either using modest algebra or by being geometric (e.g., Towers of Hanoi, or tiling problems).

Intro to Number Theory. Typical first topics, such as modular arithmetic, but going further in the second week.

Mathematical Origami. Students simultaneously learn origami and learn what shapes, lengths and angles can be constructed by origami techniques. Materials come from Tom Hull's book *Project Origami: Activities for Exploring Mathematics.*

Heavenly Mathematics. How trigonometry was invented by the ancients, led by math historian Glen van Brummelen.

Spherical Trigonometry. An introduction from scratch to this now little studied subject, and how it too was created long ago.

Analytic Geometry (2 weeks). A much more sophisticated approach than given in schools, emphasizing conics and Descartes' view of equations as being properties of curves.

AMC Practice. We have given courses on AMC8, 10, 12 and AIME. Sometimes we just work through individual tests, sometimes timed, and then discuss general techniques and mathematical ideas when comparing solutions. But for AMC10 we now use Faires' *First Steps for Math Olympians*, which groups problems by mathematical topic, thus giving much more coherence and mathematical structure to the course.

Proof by Story. Proof by combinatorial argument, by picture, and by any means that relies on using concepts rather than computations.

Taming the Torus. Hands-on experience with the torus, with students often pasting them together and cutting them apart, leading to discoveries of such thing as the coloring number.

Out for the Count, In for Infinity. An introduction to thinking about infinity, including how past generations thought about it, not always successfully.

Geometric Ramsey Theory. Questions such as: if you 2-color all the points of the plane, must some rectangle have all 4 vertices the same color?

Vector Geometry. Introduction of vectors and their application to proving geometric results in 2 and 3 dimensions.

Problem Writing. It's not so easy to come up with a great problem, one that's easy to state with a short but non-obvious proof. Prof Vandervelde, who has written hundreds of Mandelbrot Contest problems, shows that problem writing is at least partly a teachable skill, where practice and group discussion can help.

Linear Geometry. In a linear space we can talk both about convexity and set sums. (not unions). For instance, is the sum of two convex sets convex? What's the sum of a parabolic segment and itself?

Hyperbolic Geometry (2 weeks). The course begins with a discussion of Euclid's Fifth postulate and the efforts to prove it from the other four of the Euclidean postulates, and then progresses from the hyperbolic postulates to culminate in Gauss's proof of the area of a triangle in terms of angular defect.

Chessboard Problems. It is well known how chessboard coloring shows that you can't use dominoes to cover the whole chessboard less two opposite corners. Prof Watkins has written a whole book of increasingly subtle such problems, and used parts of it for this class.

Graph Theory. Basic ideas like the theorems on Euler cycles.

Puzzles and Paradoxes. Various puzzles from Smulyan's book, fascinating per se, that begin to get at issues in the foundations of mathematics.

Fair Division. Modern results in fair division, for instance, results using Sperner's Lemma. Prof Su gave this material in plenaries in 2008, and gave them in a breakout in 2009.

Plenaries forming a sequence of a week or more:

History of Mathematics (4 weeks). In the first 2 weeks Prof Van Brummelen covered from prehistoric times to Islamic Mathematics, and in the latter two Prof Drucker covered from medieval Europe to the modern day. A broad historical survey, in part emphasizing lore and personalities, but there is a more important reason for including history as a camp requirement. Students see that mathematics ideas developed in fits and starts, with different perspectives in different times, and that they did not spring into the human mind full blown the way it usually appears in textbooks.

Mathematical Writing (4 weeks). Through discussion of student solutions to the Qualifying Test problems, various nuts and bolts issues about mathematical writing come up without our lecturing on them directly (which would be ineffective). For instance, the issue of a good balance between symbols and

words comes up again and again, as well as the value of definitions, displays, labeling, and other highlighting. Larger issues are addressed as well, such as methods of proof and providing a clear overall structure to an argument.

Probability and Card Tricks (1 week). Prof Su first give a general introduction to probability and some combinatorics, and then showed off various card tricks, followed by an explanation of why they worked. He also devoted one lecture to some of the Math Fun Facts on his website.