MathPath Breakout Catalog 2017
Final, 7/14/17

Week 1, Morning

Mathematica I, Prof C (Silva Chang)

Mathematica is a popular computer software program used by mathematicians and scientists. We will learn how to use Mathematica to perform basic mathematical operations, solve equations, create lists, and generate 2D graphics. No computer programming experience is necessary. (At the end of camp, each MathPath participant will receive a free copy of Mathematica.) *1 star*

Basic Counting (Combinatorics), Prof D (Matt DeLong)

For students who know a little about how to count, but want to know more and get better. We start with the basics: sum rule, product rule, permutations, combinations, the binomial theorem. Then we learn about "combinatorial arguments", sometimes called proof by story. Time permitting, we will look at inclusion-exclusion, and pigeon-hole arguments. This course is a good prerequisite for various later courses that involve counting. *2 stars*

Induction, Ms O'Neill (Granya O'Neill)

If you line up infinitely many dominoes on their ends, with each one close enough to the previous one, and knock over the first, then all infinitely many fall down. This is the essence of mathematical induction, the main proof technique when you have infinitely many statements to prove indexed by the integers, such as

1-ring Towers of Hanoi can be won. 2-ring Towers of Hanoi can be won…. 487-ring Towers of Hanoi can be won….

Every budding mathematician needs to know mathematical induction, and it's a great proof technique to learn first, because it has a standard template (unlike most proof techniques) and yet leaves room for an infinite amount of variety and ingenuity. Thus this course is one of MathPath's foundation courses.

But don't take my word for it. Here is what a MathPath student wrote on an AoPS MathPath forum on June 24, 2010:

> I took induction last year, and I knew induction before the class. But it was very well taught, and I learned how to write proofs by induction, which was very valuable on the USAJMO. I'd recommend this class to anyone who wants to learn about induction, whether you know it at the beginning or not.

The point: Even if you have done induction before, you don't really *know* induction, because it can be used in so many ways in so many parts of mathematics. Every mathematician will probably do 1000 inductions in his/her life. Get 50 under your belt in this course. *2 stars*

Analytical Geometry, Dr T (George Thomas)

Francois Vieta's introduction of literal symbols for representing a class of numerical values was followed in 1637 by Rene Descartes' application of it in Geometry to represent a point and for the representation of lines and curves by equations. He set up an oblique coordinate system and then the rectangular coordinate system. A first degree equation in x and y gives a straight line in this system, and conversely, a straight line has a first degree equation. The discussion goes very briefly over the various forms of the equation of a straight line, and an example of how algebra is used to "analyze" geometric construction problems. The area of a triangle is obtained in two ways: In terms of the coordinates of the vertices and in terms of the equations of the three lines constituting the sides. Equations of the various conics are obtained from the Homogeneous General Equation of the Second Degree. The method of the rotation of axes is used to determine the conic an equation of the second degree represents. It is shown that given any five points in the plane there is a unique conic passing through them. Equations of tangents, normals, subtangents, and subnormals are obtained for the circle first and then for the other conics. Poles and Polars, and the "correlation" between pole and its polar for a conic are shown.

The general second degree equation in two variables is shown with its nine affine forms of which ellipse, parabola, and hyperbola are just three. The method of transformation of coordinates to convert a second degree equation to its most simplified form is illustrated. The simplified form is related – "affinis" – to the original and this leads in to a discussion of affine transformations and affine geometry. Whereas an ellipse is transformed in this manner to another ellipse and not, for instance, a hyperbola, there is a transformation, more general, that takes any conic to any other conic. It is the "projective" transformation – all conics are circles when viewed from the vertex of an infinite cone. This example is used to point out the role played by Analytic Geometry in the mathematical theory of perspective for curves: when a curve is specified an equation, the equation of the perspective view is obtainable by suitably transforming x and y. Pre-requisites: Algebra 2 and Trigonometry. *3 stars*

Elementary Graph Theory, April (April Verser)

Most people think of a "graph" as a visual representation of data - a function, assorted information, etc. In the study of graph theory, we define a "graph" to be a set of points, called "vertices", and a set of lines connecting two of those "vertices", called "edges". Graphs of this sort can be used to diagram, understand, and solve many mathematical problems; some of which may surprise you! In this breakout, we will be investigating the fundamentals of graph theory, and discovering problems that can be solved by being diagrammed as graphs. *1 star*

Polygon Differencing Games, Dr Y (Phillip Yasskin)

Put non-negative integers on the corners of a polygon and their differences at the midpoints. Connect the midpoints and repeat the process until it terminates. This is a Polygon Differencing Game. What are the final configurations? The final state is very different for a triangle and a square. So the question becomes: For which N does the N-gon Differencing Game behave like a square and which like a triangle? *2-3 stars*

Week 1, Afternoon

AMC 12/ARML, Prof C (Silva Chang)

This course will cover problem solving techniques that are useful for AMC 12 or ARML short-answer problems. The two contests feature similar problems covering topics like counting, geometry, trigonometry, logarithms, and polynomials. The ARML competition has a relay round for 3-member teams where each team member's answer to a math problem is used by another team member to solve a different problem. Each day we will practice at least one ARML relay round. Prior contest math experience is recommended. *2-3 stars*

Elementary Logic, Prof D (Matt DeLong)

If you attend MathPath, you cannot be a cow. Bessie is a cow. Most of us would correctly

deduce that Bessie cannot attend MathPath, but how do we know this, and how can we prove it? Logic serves as the foundation for our reasoning, setting out formally the rules we understand intuitively. Much as algebra allows us to generalize relationships among numbers by using variables to represent quantities, we will use symbols to represent propositions, demonstrating the form or structure of an argument (premises leading to a conclusion). Then we will formally prove these arguments using rules such as Modus Ponendo Ponens, Modus Tollendo Tollens, and Reductio Ad Absurdum. We will explore truth tables, semantic trees, symbolic logic proofs, and potentially predicate logic. Also, there are paradoxes which seem to defy common sense. We shall see how putting some paradoxical examples into the regimented forms demanded by logic causes the paradoxical element to disappear. *1-2 stars*

AMC8/Chapter Mathcounts, Ms O'Neill (Granya O'Neill)

Working on questions selected from old AMC-8 and MATHCOUNTS exams, we will study the classic problems that stump students when they near the end of these tests. While we will spend a little time making sure that we can move quickly through the easier problems, thereby saving time for the more difficult ones, our focus will be to develop strategies we can use to solve the most challenging problems on the exams. Interesting divisibility questions, overlapping circle regions, tough probability problems, algebra shortcuts, and logic conundrums will keep us busy, although we'll also make time for some Countdown practice. *1 star*

Heavenly Mathematics, Glen (Glen Van Brummelen)

How were the ancient astronomers able to find their way around the heavens without anything even as sophisticated as a telescope? With some clever observations and a little math, it's amazing how much you can infer. Following the footsteps of the ancient Greeks, we will eventually determine the distance from the Earth to the Moon...using only our brains and a meter stick. Along the way, we will develop the fundamentals of the subject the Greeks invented for this purpose, now a part of the school mathematics curriculum: trigonometry. Scientific calculators are encouraged. *2 stars*

Number Theory I, Dr V (Sam Vandervelde)

Number theory has delighted young mathematicians throughout the years due to the accessibility of its ideas and the ingenuity of its techniques. This introductory level breakout presents the foundations of the subject, starting with divisibility and moving on to the Euclidean Algorithm, primes, factoring, counting divisors, perfect numbers, and relatively prime integers. We will conclude with a first look at congruences and modular arithmetic. *1 star*

Subcollection Sum Divisibility Theorems, Dr Y (Phillip Yasskin)

The goal of this session is to determine the values of k and n for which the following statement is true or false:

> $P(k, n)$: For every collection, S, of n integers there is a subcollection, T, of k integers whose sum is divisible by k.

We will first look at examples where $P(k, n)$ is true. Next, we will determine all values of k and n for which $P(k, n)$ is false by giving counterexamples. And third we will determine all values of k and n for which $P(k, n)$ is true. *3-4 stars*

Week 2, History Pullout for Returnees

Geometrical Constructions, Prof Rogness (Jon Rogness)

The history of mathematics has been infused with geometry since the very beginning, often with surprising links to algebra and other fields; did you know you can solve a quadratic equation with a geometric construction? During this week's pullout we'll survey various historical constructions to discover connections from geometry to algebra, topology, and architecture.

Week 2, Morning

Mathematica II, Prof C (Silva Chang)

This course will expand on the topics covered in Mathematica I. We will learn how to define our own functions, how to create animations using the Manipulate command, and how to generate 3D graphics. The exercises will be more challenging than the ones in Mathematica I. *2-3 stars*

Number Theory II, Prof D (Matt DeLong)

Suppose a fruit stand owner wants to arrange oranges neatly. If the oranges are arranged in rows of 5 then there are 2 left over, if arranged in rows of 6 then there is 1 left over, and if arranged in rows of 7 there are 3 left over. How many oranges could the owner have? Is there a unique solution?

Answering this question requires a deeper understanding of modular arithmetic than we achieve in NT1. So in NT2 we discuss inverses mod m, the Chinese Remainder Theorem, Fermat's Little Theorem, Wilson's Theorem, the Euler phi-function and Euler's extension of Fermat's Little Theorem. These results will allow us to answer other questions as well, such as: How many ways can 1 be factored? What happens when one repeatedly multiplies a number by itself? And because the answers to such questions are so interesting, we can all enjoy quick divisibility tricks. *2 stars*

Proof by Story, Ms O'Neill (Granya O'Neill)

Many identities involving, say, binomial coefficients or Fibonacci numbers that are commonly proven by induction or manipulating algebraic expressions also have beautiful bijection proofs. Such bijection proofs are often called *proof by story*, since we come up with two different ways ("stories") to count the same thing. These proofs often give insight into why the identity holds, and are so elegant that you will be convinced that there could not be a better proof of that particular theorem. Paul Erdos said that such proofs were from "The Book."   In this breakout, we will devise as many proofs by story for identities involving binomial coefficients and Fibonacci numbers as we can. 2-*3 stars*

Cryptology, Prof Rogness (Jon Rogness)

Cryptology is the art of creating – and breaking! – ciphers which take information and make it secret. We'll begin by looking at basic codes, and using modular arithmetic to explain how certain ciphers can be described mathematically. We'll also examine the Vigenere Cipher, which was described as "unbreakable" just 100 years ago, but is nearly as easy for mathematicians to break as a simple "Cryptoquip" in the newspaper. By the end of the week we'll cover modern cryptosystems, including methods used by websites like Facebook and Amazon to protect passwords and credit card numbers. Knowledge of modular arithmetic, for instance from Number Theory I or the equivalent, will be used throughout the week. Additional number theory will be introduced as needed, so prior knowledge of additional number theory is helpful but not necessary. *2 stars*

Hyperbolic Geometry, Dr T (George Thomas)

This is an ideal course for the future mathematician in that it combines history and the axiomatic method. The course begins with a discussion of Neutral Geometry – axioms, the Exterior Angle Theorem, Alternate Interior Angle theorem and the Saccheri-Legendre Theorem. Next: Euclid's

Fifth postulate ("Fifth") is added; its equivalence with the Euclidean Parallel Postulate and the Converse of the Alternate Interior Angle Theorem is shown.

The efforts of two millennia to prove the Fifth from the Neutral Geometry postulates and the introduction of the hyperbolic postulate. H. Liebmann's proof: The area of a singly asymptotic triangle is finite. The finiteness of triangular area and Gauss's proof of the area of a triangle in terms of angular defect. The Bolya formula connecting the angle that a perpendicular of given length to a given line makes with a parallel to the given line is derived. Bolya's construction of ultraparallels. The Klein, Poincare and Beltrami models. ! *Pre-requisites: Euclidean Geometry, Trigonometry. 3 stars*

Four Star Problem Solving, Dr V (Sam Vandervelde)

This breakout is designed for students with substantial contest problem solving background who have a solid working knowledge of algebra, geometry, counting, and number theory. The purpose is to gain experience with a variety of advanced tools and techniques within these areas that often surface in contests such as the AMC-10. We will examine each topic in turn, culminating in a class contest on the last day to apply what we've learned. 4 *stars*

Week 2, Afternoon

State/National MATHCOUNTS, Prof C (Silva Chang)

This course will cover problem solving techniques that are useful for state and national MATHCOUNTS. Topics will include number theory, algebra, geometry, and combinatorics. Each day will include a Countdown Round practice. Prior contest math experience will be helpful. *2 stars*

Knot Theory, Prof D (Matt DeLong)

Knots have fascinated farmers, sailors, artists, and myth-makers since prehistoric times, both for their practical usefulness and for their aesthetic symbolism. For over 200 years, mathematicians have intensely studied knots, both for their own intrinsic mathematical interest and for their potential application to chemistry, physics and biology. Today, Knot Theory is an active field of mathematical research with many important applications. It is visual, computational and hands-on, and there are many easily stated open problems.

This class will give an introduction to the fundamental questions of Knot Theory. It will take students from simple activities with strings to open problems in the field. It will answer such questions as, "what is the difference between a knot in the mathematical sense and a knot in the everyday sense?," "how do I tell two knots apart?," "how can I tell whether a knot can be untangled?," and "how many different knots are there?" *2 stars*

Geogebra, Ms O'Neill (Granya O'Neill)

Geogebra is an interactive geometry, algebra, statistics, and calculus program developed by mathematicians and engineers for teaching and learning mathematics. In this breakout, students will learn to use Geogebra to construct a wide variety of sketches. The course assumes no prior experience with Geogebra but will move quickly from basic models to more advanced ones as students become familiar with the many tools which are part of the software. We will start by learning how Geogebra mimics traditional construction using straightedge and compass and then consider applications in analytic geometry. In addition to illustrating and exploring ideas from Euclidean geometry, however, we will also investigate concepts from algebra, trigonometry, and calculus. Our emphasis will be on applications not easily performed on a graphing calculator. During the week, we will draw many beautiful and interesting sketches including spirals (Fibonacci and Theodorus), fractals, and tessellations. *2 stars*

Spherical Trigonometry, Glen (Glen Van Brummelen)

To find your way through the heavens, along the earth, or across the oceans, you need mathematics. But the math you learn in school mostly takes place on a flat surface --- not the sphere of the heavens, or the earth. To navigate properly, we develop a completely new trigonometry that allows us to find our way around a sphere. We shall discover surprising symmetries and a rich world of theorems, many of which are beautiful and unexpected extensions of some of the most familiar geometric theorems we have learned in school. Scientific calculators are encouraged. *3 stars*

Elegant Area, Dr V (Sam Vandervelde)

There exists a remarkably diverse collection of formulas for computing the area of various regions within the Euclidean plane. We are all familiar with one-half base times height; during this breakout we will encounter at least ten triangle area formulas, along with a selection of other formulas that give the areas of quadrilaterals and beyond. We will derive some of the formulas, see novel applications of others, and create our own original area problems. Students taking this breakout should be comfortable with algebra and have completed a basic introduction to geometry. *3 stars*

Count it Like Polya, Dr Z (Paul Zeitz)

How many different ways can the edges of a cube be colored using three colors? It's certainly less than 3^12, because so many arrangements are overcounted due to symmetry. We will study the systematic method of "counting modulo symmetry," and learn some group theory and generatingfunctionology along the way. Prerequisite of Basic Counting (Combinatorics). *2 stars*

Week 3, History Pullout for Returnees

Convergence of Real Number Sequences and Construction of Real Numbers, Dr T (George Thomas)

One of the great successes of Greek geometry is the Method of Exhaustion, which involved the determination of the area of a figure as the sum of a sequence of areas of polygons in to which the figure could be dissected. The greatest example is Archimedes's determination of the area of a parabolic segment as 4/3 times the area of a certain triangle in that figure.

The Method of Exhaustion was connected to specific figures and the reasoning used in a specific context could not be used in others. Consequently mathematicians searched for new methods that had more general application. This led to the consideration of "Series" and Continued Fractions. After all, Archimedes's quadrature of the parabola uses the series $\sum_0^\infty 4^{-n}$.

Earliest works on series included those of Viete, Gregoire de Saint-Vincent, Wallis, and James Gregory. The idea of sum lies at the heart of the concept of series. For example, Saint-Vincent considers the famous paradox of Achilles and the Turtle and showed that Achilles gains on the turtle according to a geometric series, which has a finite sum.

Later we see manipulation of infinite series by Liebniz, Newton, Bernoulli, Taylor, Euler, and

Fourier. Particularly in the case of Euler we see that the goal was the sum; convergence issues were not a formal consideration. The modern theory of series may be said to date from the publication of a paper by Gauss in 1812 and Cauchy's book, *Analyse Algebrique*, in 1821.

Week 3, Morning

Monty Meets Bayes, Silas (Silas Johnson)

Will you choose Door #1, #2, or #3? But wait, behind Door #2 is... a goat! Still think your door has the car? Or do you want to switch? The Monty Hall Problem, loosely based on the 1960's game show Let's Make A Deal, is a famous "paradox" of probability. Although the answer is perfectly straightforward, it's counterintuitive enough that it at one point it spawned heated debate in magazine editorial pages; even professional mathematicians lost their cool defending their (incorrect!) opinions. Using the Monty Hall Problem and several variations as our starting point, we'll begin by looking at some of the basic ideas in probability. Then we'll redo it all through the lens of Bayes' Theorem, an extremely important tool for theoretical and practical mathematicians alike. *1-2 stars*

Exotic Arithmetic, Dr R (Harold Reiter)

Representing integers and fractions in unusual ways, and the arithmetic that follows. Fractional Base, Fibonacci, negative 4, Cantor's representation. Also, some modular arithmetic as time allows. *1-2 stars*

Integer Partitions, Dr. J (James Sellers)

How many ways can you write 4 as a sum of positive integers? If order matters, so that we consider 3+1 and 1+3 to be different, then the answer (8) is an easy application of very basic combinatorics. But if order of the parts doesn't matter, then the answer is 5. The partitions in question are 4, 3+1, 2+2, 2+1+1, and 1+1+1+1. This simple question leads to a rich, beautiful, and surprisingly challenging mathematical area which overlaps greatly with numerous areas of mathematics, including algebra, combinatorics and number theory. We will look at integer partitions from many different perspectives, including elegant bijections, cool recursions, divisibility properties of partition values (a la Srinivasa Ramanujan and G. H. Hardy who were recently featured in "The Man Who Knew Infinity"), and some wonderful algebra connections known as generating functions. For this breakout, some previous experience with elementary combinatorics (sum rule, product rule, permutations, combinations, the binomial theorem) and experience with manipulation of polynomials and geometric series will be helpful. *3-4 stars*

Topics in Advanced Graph Theory, Kip (Kip Sumner)

We will study Ramsey Numbers, from the basic idea through to an exploration of how the search continues for larger ones, and we'll prove an important theorem that helps narrow that search. There will be colouring tasks which underpin a greater understanding of the concept. We will also study two theorems associated with Hamiltonian Cycles in graphs, one providing a sufficient condition and the other a necessary condition for the existence of a HC in a graph. The climax of the week will be a close look at a graph devised by William Tutte in 1946 to refute Tait's

Conjecture. (A street was named this year for Bill Tutte in Waterloo, Ontario; creating a nice symmetry with Alan Turing Way in Manchester, England.) Prerequisite: some previous experience with graph theory, such as from taking the Elementary Graph Theory Breakout. *2-3 stars*

Elliptic Geometry, Dr T (George Thomas)

In this course we explain why there are exactly three 3-dimensional constant-curvature geometries – Euclidean, hyperbolic and elliptic. Whereas the Euclidean and hyperbolic geometries are neutral geometries, we show that Elliptic Geometry is not a Neutral Geometry. However, Elliptic Geometry complements the Euclidean and Hyperbolic geometries in terms of the number of parallels, namely none. We discuss Elliptic Geometry using two models – Spherical Geometry and the Klein Circle model – to show also that it is a projective space with an elliptic metric. We also prove some fundamental properties of a sphere including that a geodesic is a great circle on the sphere. *Pre-requisite: Trigonometry. Those who took Spherical Geometry will find some overlap due to the use of the sphere in building the model of elliptic Geometry. 3 stars*

Surfaces and silly straws, Cornelia (Cornelia Van Cott)

In this breakout course, we dive into the wild and wonderful world of surfaces – starting with the sphere and torus and moving on to more exotic surfaces, too! Our main goal will be to understand a famous (and beautiful) theorem called The Classification of Surfaces. Along the way, we will learn how to cut up surfaces into triangles (a process called triangulation), we'll learn how to determine the Euler characteristic and orientability of surfaces, and more. There will not be much algebra in this breakout, but you should enjoy thinking about shapes in 3-dimensions. [You might be saying to yourself, "Wait, what are silly straws, and how did they get into the course title?" Take this breakout course, and you'll find out!] *2 stars*

Week 3, Afternoon

Primality Testing, Keith (Keith Conrad)

When Gauss wrote in 1801 that "The problem of distinguishing prime numbers from composite numbers and of resolving the latter into their prime factors is known to be one of the most important and useful in arithmetic," he did not know that 200 years later this problem would become very significant for cryptography: it is used every day by millions of people, usually without realizing it. We will discuss how to check the primality of integers by deterministic and probabilistic algorithms. The audience should be familiar with modular arithmetic as covered in the course Number Theory I and Number Theory II, including Fermat's little theorem. *3-4 stars*

When the Impossible is Provable, Coach D (Tom Drucker)

It is common to claim that nothing is impossible. Whatever one thinks about that belief, there is no doubt that mathematics can prove that some problems within mathematics are impossible to solve. In order to understand why they're impossible, you have to understand the rules under which the problem has to be solved if it's to count as soluble. We'll look at four problems where one can claim that the solution is impossible, although we shall try to figure out what is still possible. The four problems are the trisection of an angle, the solution of a fifth-degree equation, the incompleteness of arithmetic, and the unsolvability of Diophantine equations. You will have heard something about all of these in the history lectures, but in this course we shall try to figure out if there is a common element in all the situations where we claim that something is 'impossible'. No specific background is required, but it would help if you've seen some trigonometry and had plenty of practice with algebra. *3 stars*

Special Relativity: The Mathematics of Paradox, Silas (Silas Johnson)

Einstein's Special Theory of Relativity, at heart, is not about physics so much as the geometry of the 4-dimensional space-time we live in. Motivated by the paradoxes that plague older ways of thinking about physics, we'll discover the equations that define this geometry. We'll finish by using these all-important equations to resolve the paradoxes we explored earlier in the week. If we have extra time, we might take a look at why faster-than-light travel is impossible. *3 stars*

Counting with Cubes, Dr R (Harold Reiter)

You can see three faces of a 4 x 5 x 6 rectangular prism made from 120 unit cubes. How many cubes can you actually see? In this course we'll learn how to use the inclusion/exclusion principle, complimentary counting, and other fundamental ideas of combinatorics. *2-3 stars*

Thank You, Fibonacci!, Dr. J (James Sellers)

More than 800 years ago, Leonardo of Pisa single-handedly began the study of recurrence relations in the Western world with his introduction of the Fibonacci numbers. We will consider these numbers from a wide variety of perspectives, primarily exploring various properties of this wonderful sequence of numbers. We will be led to consider divisibility properties of the Fibonacci numbers, numerous connections to algebra (including developing the closed-form formula for the Fibonacci numbers which completely explains the mathematical origins of the golden ratio), and much more! We will look at the Fibonacci numbers through the lens of a special family of 2x2 matrices, and we will see how they are related to colorful tilings of a rectangular strip. We will also generalize much of what we learn about the Fibonacci numbers to other recurrent sequences (in a very natural way), expanding our understanding of such sequences. Some experience with manipulation of polynomials will be helpful. *2-3 stars*

Frobenius numbers and chicken nuggets, Cornelia (Cornelia Van Cott)

If you're vegetarian or vegan, you may not think that a course about chicken nuggets sounds all that enticing. But think again! This topic has attracted interest from mathematicians of all culinary preferences since the 1800's, and mathematicians are still working on the problem today. Here is how the problem starts off: McDonald's sells chicken nuggets in packs of 6, 9, and 20. What is the largest number of chicken nuggets that one cannot purchase? The generalizations of this question lead to a series of elegant results and open questions. Prerequisites: Number Theory I. *3-4 stars*

Week 4, Morning

Wallpaper Patterns and Life on the Klein Bottle, Prof Cahn (Patricia Cahn)

What is it like to live on a Donut? A Mobius Strip? A Klein Bottle? We'll think about this using wallpaper patterns—repeating patterns in the plane with nice symmetries. We'll learn how mathematicians classify these patterns, and practice telling two wallpaper patterns apart with a quick glance. Then we'll fold the patterns up to produce lots of interesting surfaces, called orbifolds, and study life in these spaces. There are no specific prerequisites for this class. *3 stars*

Combinatorial Games, Dylan (Dylan Hendrickson)

Let's play a game. We start with some number of piles of stones, and take turns removing some stones. On your turn, you pick a pile and remove any (positive integer) number of stones from that pile. Whoever takes the last stone wins. This simple game is called Nim, and the winning strategy is surprising and beautiful. Amazingly, a large class of games turn out to be equivalent to certain positions in Nim.

In this class, we'll learn strategies for finding winning strategies in games. The focus will be on analyzing a class of games known as impartial combinatorial games, and we'll see how they all reduce to Nim. We'll then figure out how to win Nim, and thus also any impartial combinatorial game. If we have time, we might talk about strategy-stealing arguments, which can show that someone has a winning strategy without knowing what that strategy is. *4 stars*

Cryptology, Silas (Silas Johnson)

See description in Week 2 morning. *2 stars*

Coloring with Polynomials, Dr Rana (Julie Rana)

A graph is a set of vertices connected by edges. Given a specific graph, how many colors do I need in order to guarantee that I can color the vertices in such a way that no two adjacent vertices are the same color? This question motivated one of the most well-known theorems in graph theory: the four color theorem. We'll play a little with this theorem, then shift focus and discover ways to model graph-colorability using polynomials and finite fields. Along the way, we'll see that this gives a fascinating, albeit unwieldy(!), method for solving Sudoku puzzles. Prerequisite: Number Theory II. *3 stars*

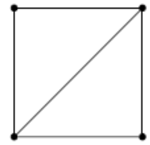
AIME Problems, Dr R (Harold Reiter)

Standard strategies for solving AIME problems. Symmetry, position notation, area model, recursion, inclusion\exclusion, and other ideas. *3 stars*

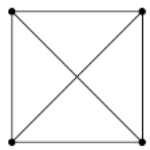
Planar Graphs (and the lives of three dimensional solids in Flatland), Prof GS (Gabriel Sosa)

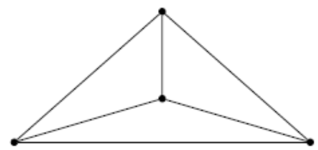
A graph is a collection of points (called vertices) and edges (line segments drawn between vertices). A graph is said to be planar if there is a way to draw it without having edges crossing.

For example, a square with one of its diagonals drawn is a planar graph with four vertices and five edges.

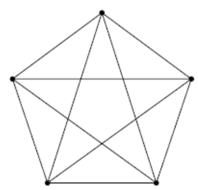


A square with its two diagonals is also a planar graph, because as a graph it is the same as the figure to its right:





On the other hand the graph below is not planar



In this break out we will study, and explore, several properties satisfied by planar graphs and the relation between planar graphs and three dimensional solids.

Aided by our knowledge of planar graphs, counting techniques and some arithmetic, we will be able to draw unexpected representations of three dimensional solids on pieces of paper and discuss how many different types of regular (and semi-regular) solids exist. *2-3 stars*

It Slices, It Dices, Cornelia (Cornelia Van Cott)

Your friends are visiting, and it’s time to slice up the pizza. What is the maximum number of pieces that you can cut a pizza into with n straight line cuts? We call such numbers pizza numbers. We will discover lots of connections between pizza numbers and other familiar mathematical objects. And of course, we will then generalize this problem, looking at higher dimensions and considering a variety of related slicing and dicing problems. *2 stars*

Week 4, Afternoon

Biological Systems Modelling, George (George Abraham)

This course is a weeklong introduction to the mathematical techniques of biological systems modelling. From a mathematical standpoint, we will be covering topics such as physical interpretation of ordinary differential equations, algebraic techniques for solving simple first and second order differential equations, second order transfer functions, and pole zero diagrams. From a physical standpoint, models we investigate will be linearized approximations of biologically inspired phenomena, including viral models, organ dynamical models, metabolic models, and more. Lectures will be interactive, supplemented with animations, system visualizations, and daily handouts the students can take notes on. Prerequisites: conceptual knowledge of the derivative and, ideally, basic computation laws (power rule, chain rule). Those who do not have the derivative computation background will be required to attend a mini lecture on derivatives Sunday night before classes start. *4 stars*

Chess and Life, Coach D (Tom Drucker)

Bobby Fischer, the late world chess champion, said, 'Chess is Life.' While that may explain some of Fischer's behaviour, it is not true that the game of chess is the game of life. In this course we'll look at some of the ways in which mathematical ideas arise out of chess and the chess board. We'll start off with a quick review of the rules of the game, although it helps if you've already seen the basics. Then we'll look at some of the interesting geometric and combinatoric problems that arise from the ordinary chessboard as well as more exotic versions (like cylindrical chess and toroidal chess). There are features associated with the rules that guarantee that games can always come to an end, but sometimes the end can be a long time coming, and we'll calculate how long a chess game can be. We'll look at some of the reasoning involved in retrograde chess problems, especially as they appear in the work of Raymond Smullyan, a mathematician who died this year. Finally, we'll see how game theory is applied differently to chess and to games where you don't know what your opponent is doing. *1-2 stars*

Simple Models of Computation, Jordan (Jordan Hines)

Suppose we wanted to write a program to determine if an integer is a multiple of 3. It's pretty clear that this is an easy task for a computer. But it turns out that there are some things that you can't write a program for, no matter how clever you are, and we can prove it! In order to approach such a proof, we need to ask - what is a program? How can we describe computation precisely? In this breakout, we'll talk about some simple mathematical models of computation and prove things about them! While our focus will be on models that are not as powerful as programming, the ideas we discuss will be closely related to the ideas found in more powerful models.

We'll start by discussing deterministic and nondeterministic finite automata, abstract machines that move into different "states" based on their input. Then, we'll introduce a tool called regular expressions, which describe strings using a small collection of symbols. We'll show that these models are not quite as different as they may seem! We'll also prove the pumping lemma, which will allow us to prove that there are some things that these models of computation cannot do! If we have time, we'll discuss some more powerful models, including the Turing machine.

Note: While the topics in this class are relevant to computer science, you don't need any knowledge of programming to take this class! *3 stars*

Special Relativity, Silas (Silas Johnson)

See description in Week 3 afternoon. *3 stars*

The Shape of Space, Dr Rana (Julie Rana)

This breakout will explore ideas in the area of math known as topology, where a donut (torus) and coffee cup are equivalent objects. It will provide a nice followup for any students who have done the previous breakouts this year which covered the torus, although those courses are not prerequisites for this one.

We'll start by learning a few basic rules in topology. Then we'll look at the torus, either as a review or brief introduction depending on whether students were in those previous breakouts. Then we'll look at how to construct other two dimensional surfaces, like the Mobius strip and Klein Bottle. As it turns out, in one of the triumphs of mathematics, we can describe how to build all of the so-called "closed surfaces" by sewing together spheres, Moebius strips and donuts. After we deal with surfaces, we'll have the necessary skills to move on to three dimensions. That means we can think about different possible shapes for the universe -- i.e. the Shape of Space -- and explain why the picture below with the dodecahedra might be a model of how our universe is put together.

There won't be much in the way of computations or algebra in this breakout, but you should like thinking about three dimensional shapes. (For example, if you like looking at nets and figuring out what the resulting shape is, we'll be doing more complicated versions of that process.) *2 stars*

KenKen, Dr R (Harold Reiter)

Strategies and techniques for solving these alluring puzzles. No Op puzzles, prime puzzles. Learn to create your own puzzles. *2 stars*

Projective Geometry, Dr T (George Thomas)

Euclidean Geometry is the geometry of lines and circles: its tools are the straight edge (unmarked ruler) and compass. There is a geometry where its constructions need only a straight edge. In this geometry a straight line joins two points and two lines never fail to meet. It turns out to be simpler than Euclid's but not too simple to be interesting. This is Projective Geometry which turns out to be the mother of all geometries, for Euclidean, Elliptic, Hyperbolic and Minkowskian Geometries are but special cases of it.

The course begins with Pappus's ancient theorem. The discussion then proceeds as follows: The new Geometry's axioms, the principle of duality, the concept of projectivity, Harmonic sets, The Fundamental Theorem of Projective Geometry, and Desargue's Theorem. *Pre-requisites: None. 3 stars*

Metrics on the plane, Cornelia (Cornelia Van Cott)

We usually define the distance between two points to be the length of the straight line connecting the points. This is the Euclidean metric. But who says it must be done this way? Why not try something else? We will investigate a variety of other options: the taxicab metric, the elevator metric, the post office metric, and more. In the process of our investigation, we will discover new geometries on the plane and understand the familiar Euclidean geometry even better. *2-3 stars*

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